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Tolerance Design of Active RC Filters

Alexander Khludenev
Department of Industrial Electronics and Information and Measuring Engineering
Orenburg State University
Orenburg, Russian Federation
avhludenev@yandex.ru

***Abstract*—An engineering technique for the tolerance design of active RC filters using advanced OrCAD PSpice tools is considered. The implementation of the tolerance synthesis method based on sensitivity analysis is substantiated. The issues of the operability margin formation and the measuring of performance parameters, their sensitivity to variations in the element parameters are discussed. The variance balance allocation allows for significant differences in the availability of discrete resistors and capacitors with the required tolerances. Variations in the operational amplifier parameters are also taken into account.**

Keywords—active RC filter, frequency magnitude response (FMR), design specification, acceptability region, operability margin, element tolerance, tolerance region, production yield, probability distribution, worst case allocation, statistical allocation, sensitivity analysis, gain bandwidth product (GBW)

I. INTRODUCTION

Analog devices are significantly inferior in comparison with digital ones in terms of achievements in synthesis automation. Active RC filters are one of the few exceptions. This is confirmed by specialized software for automatic design of active RC filter circuits, e.g., FilterLab (Microchip), FilterCAD (Linear Technology), FilterPro Desktop (Texas Instruments).

Unfortunately, these and other similar tools are unable to assign tolerances to circuit elements to ensure the frequency magnitude response (FMR) of the manufactured filters meets the specifications at an acceptable percent yield. The Analog Filter Wizard design tool (Analog Devices) allows the user to interactively assign element tolerances and estimate FMR dissipation. At the same time, the selection of tolerance combination is time-consuming and does not guarantee a successful solution to the problem.

General purpose Electronic Design Automation (EDA) systems provide powerful analysis and parametric verification tools as well as parametric optimization tools. The author is not aware of the capabilities of these EDA tools to calculate element tolerance values. EDA users define tolerance values through time-consuming interactive optimization. There is no guarantee of achieving a result that meets the specification constraints.

Various methods for tolerance design have been developed [1], [2], [3]. Assessment of their applicability for the parametric synthesis of active RC filters in the EDA environment is an urgent task. This paper addresses the problem of developing an engineering technique for the active RC filters tolerance synthesis, suitable for use a general-purpose EDA tool. The choice of methods for tolerance allocation was made taking into account the available tools provided by OrCAD PSpice Designer v.17.2 [4]. The technique should allow:

1) Direct calculation of the initial approximation of the tolerance values;

2) A compliance assessment of the result obtained with the feasibility constraints.

A suitable initial approximation will greatly simplify the allocation of standard tolerances. If the feasibility constraints are not met, the designer can modify the problem conditions by optimizing the schematics and parameter values.

II. PROBLEM FORMALIZATION

The filter design specifications are determined by the constraints imposed on the transmission gain $H(f)$ in the passband:

$$H_{PMIN} \leq H(f, \boldsymbol{X}) \leq H_{PMAX}, \tag{1}$$

and in the stopband:

$$H(f, \boldsymbol{X}) \leq H_{SMAX}, \tag{2}$$

where H_{PMIN} and H_{PMAX} are minimum and maximum permissible values of the passband gain, H_{SMAX} is maximum permissible values of the stopband gain, f is frequency, $\boldsymbol{X} = \{x_1, x_2, \ldots, x_m\}$ are circuit element parameters. Permissible passband ripple (or flatness) $\Delta H_P = H_{PMAX} - H_{PMIN}$.

The feasible region is an area in the space of performance parameters where constraints (1) and (2) are fulfilled. It is mapped into the $\boldsymbol{X}$ parameter space to form the acceptability region. The fact values $\boldsymbol{X}$ are distributed randomly around the nominal values $\boldsymbol{X}_0 = \{x_{01}, x_{02}, \ldots, x_{0m}\}$ within the absolute manufacturing tolerances $\Delta x_{max\,i}$:

$$x_{0i} - \Delta x_{max\,i} \leq x_i \leq x_{0i} + \Delta x_{max\,i}. \tag{3}$$

Inequation (3) defines a tolerance area. The tolerance synthesis ensures finding the relative tolerances:

$$t_i = \Delta x_{max\,i} / x_{0i}, \tag{4}$$

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for which the tolerance area is completely within the acceptability region. To reduce the total cost of the elements, one should strive to find the maximum possible values of the relative tolerances. For a solution to the problem of tolerance synthesis to exist, the parameter nominal values $\boldsymbol{X}_0$ should not be located on the boundaries of the acceptability region. This requirement provides operability margins in the passband for constraint (1):

$$a_P(f) = min(H(f, \boldsymbol{X}_0) - H_{PMIN}, H_{PMAX} - H(f, \boldsymbol{X}_0)), \qquad (5)$$

and the stopband for constraint (2):

$$a_S(f) = H_{SMAX} - H(f, \boldsymbol{X}_0)). \qquad (6)$$

Operability margins should be ensured when synthesizing the filter nominal transfer function by tightening the requirements for the passband ripple height:

$$\Delta H_P(\boldsymbol{X}_0) = max(H(f, \boldsymbol{X}_0)) - min(H(f, \boldsymbol{X}_0)) < \Delta H_P, \qquad (7)$$

and for the maximum value of $H(f, \boldsymbol{X}_0)$ in the stopband:

$$H_{SMAX}(\boldsymbol{X}_0) = max(H(f, \boldsymbol{X}_0)) < H_{SMAX}. \qquad (8)$$

Centering frequency response $H(f, \boldsymbol{X}_0)$ in the passband between the boundaries H_{PMIN} and H_{PMAX} is performed to maximize the minimum margin $a_{PMIN} = min(a_P(f))$ in (5). The values of the operability margins determine the conditions for the tolerance synthesis.

The formation of the operability margins a_{PMIN} and a_{SMIN} for the Chebyshev bandpass filter is explained in Fig. 1. The passband cutoff frequencies are denoted f_{PL} and f_{PH}, the stopband edge frequencies are f_{SL} and f_{SH}.

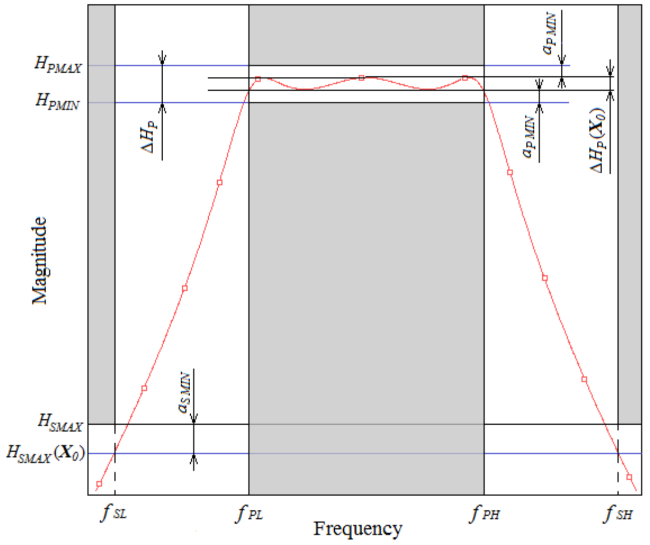

Fig. 1. Specifications and FMR of the Chebyshev bandpass filter.

III. APPLICABLE TOLERANCE ALLOCATION METHODS

When choosing methods for tolerance synthesis, the capabilities of the EDA tools used must be taken into account. The PSpice Advanced Analysis modules provide a worst-case and Monte Carlo tolerance analysis tool as well as a sensitivity analysis tool. The parametric optimization tool can't search for tolerance values. Therefore, it is advisable to use analytical methods based on sensitivity analysis.

The deviation of the performance parameter $y(\boldsymbol{X})$ with infinitesimal deviations of m element parameters is determined by the total differential:

$$dy = \sum_{i=1}^{m} \frac{\partial y(\boldsymbol{X})}{\partial x_i} \cdot dx_i. \qquad (9)$$

For the relative deviations of the parameters:

$$dy = \sum_{i=1}^{m} S_{x_i}^{y}(\boldsymbol{X}) \cdot dx_i / x_i, \qquad (10)$$

where $S_{x_i}^{y}(\boldsymbol{X}) = (\partial y(\boldsymbol{X})/\partial x_i) \cdot x_i$ is semi-relative sensitivity of y to a change in the parameter x_i.

Suppose that changes in sensitivity values within the acceptability region can be neglected, and using their values at the reference point $\boldsymbol{X}_0$ provides an acceptable method error. Then the deviation of the performance parameter with finite deviations Δx_i of m element parameters relative to the nominal values x_{0i}:

$$\Delta y = \sum_{i=1}^{m} S_{x_i}^{y}(\boldsymbol{X}_0) \cdot \Delta x_i / x_{0i}. \qquad (11)$$

Let's denote the relative deviations $\delta x_i = \Delta x_i / x_{0i}$. The δx_i values are random variables limited by production tolerances $|\delta x_i| \le t_i$.

A. Worst Case Allocation

Worst-case tolerances can be obtained by assuming:

$$\Delta y_{max} = \sum_{i=1}^{m} |S_{x_i}^{y}(\boldsymbol{X}_0)| \cdot t_i. \qquad (12)$$

Assuming that Δy_{max} is the available operability margin, with the equal values of element tolerances:

$$t_i = \Delta y_{max} / \sum_{i=1}^{m} |S_{x_i}^{y}(\boldsymbol{X}_0)|. \qquad (13)$$

Otherwise, assuming that all elements have the same contribution to Δy_{max}:

$$t_i = \Delta y_{max} / (m \cdot |S_{x_i}^{y}(\boldsymbol{X}_0)|). \qquad (14)$$

Worst-case tolerance estimates may be unnecessarily pessimistic. The use of these estimates may lead to unjustified overstatement of the product value.

B. *Statistical Allocation*

If the element manufacturing technology does not use trimming and sorting into groups according to tolerance values, the deviation values δx_i are well approximated by the normal distribution law with zero mean and variance $\sigma_{\delta xi}^2 = t_i^2/9$. When manufacturing precision elements, the sorting out according to tolerance values is performed. The probability density function can be approximated by a uniform law with variance $\sigma_{\delta xi}^2 = t_i^2/3$. It is possible to establish the ratio $\sigma_{\delta xi} = l_i \cdot t_i$.

If the number of discrete elements m is large and deviation values δx_i are statistically independent, then the probability distribution Δy is well approximated by the normal law with zero mean and variance [5]:

$$\sigma_{\Delta y}^2 = \sum_{i=1}^{m} (S_{x_i}^{y}(\boldsymbol{X_0}))^2 \cdot \sigma_{\delta x_i}^2 \ . \qquad (15)$$

Based on the planned manufacturing yield and the properties of the normal distribution, it is possible to choose the required ratio $\sigma_{\Delta y} = k \cdot \Delta y_{max}$. In particular, k = 1 for 68% manufacturing yield, k = 1/2 for 95%, k = 1/3 for yield 99.8%. Assuming the same contribution of the element parameter deviations to the performance parameter deviation and considering tolerance-sigma ratio, as a result:

$$t_i = k \cdot \Delta y_{max} / (\sqrt{m} \cdot l_i \cdot |S_{x_i}^{y}(\boldsymbol{X_0})|) \ . \qquad (16)$$

C. *Variance Balance Allocation*

The engineering practice should consider the market availability of components with the required tolerances. Digi-Key database analysis shows that the manufacturer group offers SMD resistors with at least 0.1% tolerance. But SMD ceramic capacitors are available with a tolerance of at least 1%.

For (12): if only parameter deviations of the passive components are taken into account, then designating the number of resistors in the circuit m_R and the number of capacitors m_C:

$$\Delta y_{max} = \sum_{i=1}^{m_R} |S_{R_i}^{y}(\boldsymbol{X_0})| \cdot t_{R_i} + \sum_{i=1}^{m_C} |S_{C_i}^{y}(\boldsymbol{X_0})| \cdot t_{C_i} . \qquad (17)$$

Assigning the available values of the capacitor tolerance t_{Ci}, the resistor tolerance values can be found:

$$t_{R_i} = (\Delta y_{max} - \sum_{i=1}^{m_C} |S_{C_i}^{y}(\boldsymbol{X_0})| \cdot t_{C_i}) / (m_R \cdot |S_{R_i}^{y}(\boldsymbol{X_0})|) . \qquad (18)$$

Similarly for (15):

$$\sigma_{\Delta y}^2 = \sum_{i=1}^{m_R} (S_{R_i}^{y}(\boldsymbol{X_0}))^2 \cdot \sigma_{\delta R_i}^2 + \sum_{i=1}^{m_C} (S_{C_i}^{y}(\boldsymbol{X_0}))^2 \cdot \sigma_{\delta C_i}^2 , \qquad (19)$$

$$t_{Ri} \approx \frac{\sqrt{(k \cdot \Delta y_{max})^2 - \sum_{i=1}^{m} (S_{ci}^{y}(\boldsymbol{X}_0))^2 \cdot (l_{ci} \cdot t_{ci})^2}}{\sqrt{m_R} \cdot l_{Ri} \cdot |S_{Ri}^{y}(\boldsymbol{X}_0)|} . \qquad (20)$$

If too low or negative t_{Ri} values are obtained, the problem has no feasible solution for the assigned t_{Ci} values. Similarly to capacitors, the available tolerance values can be assigned to high sensitivity resistors.

IV. IMPLEMENTATION

A. *Performance Parameters*

It's necessary to select correctly the performance parameter y. Sensitivity values and therefore t_i values will depend on frequency f using $y = H(f)$. The performance parameter y should not change its value in passband and stopband. These requirements are satisfied by the peak-to-peak ripple of the frequency response in the passband:

$$y_P = max(H(f, \boldsymbol{X})) - min(H(f, \boldsymbol{X})), \qquad (21)$$

and the maximum magnitude in the stopband:

$$y_S = max(H(f, \boldsymbol{X})). \qquad (22)$$

The risk of one-sided crossing of the lower or upper boundary in (1) at $y_P < \Delta H_P$ is not dangerous, since the statistical distribution Δy is close to symmetric.

B. *Sensitivity Analysis*

PSpice Advanced Analysis tools allow users to calculate semi-relative sensitivity for y_P and y_S. One factor at a time (OAT) method of the sensitivity analysis is implemented. It's based on a positive change in the parameter value of one component:

$$\Delta x_i = x_{0i} \cdot (1 + S_v \cdot t_{pi} / 100\%), \qquad (23)$$

where $t_{pi} = t_i \cdot 100\%$ is percentage tolerance, S_v is parameter variation factor (default $S_v = 0.4$). Calculations determine the measurement change between simulations with the component parameters first set at its nominal values $\boldsymbol{X_0}$ and then x_i changed by (23) $\boldsymbol{X}'_0 = \{x_{01}, ..., x_{0i} + \Delta x_i, ..., x_{0m}\}$. Sensitivity analysis tool interpolates the measured value at 1% tolerance:

$$S_{x_i}^{y}(\boldsymbol{X}_0) = [y(\boldsymbol{X}'_0) - y(\boldsymbol{X}_0)] / (S_v \cdot t_{pi}). \qquad (24)$$

Deformation of the frequency response can lead to gross errors in the calculation of the sensitivity values. An explanation is shown in Fig. 2. The red curve is the frequency response with nominal values $\boldsymbol{X}_0$, and the blue curve is the frequency response with one percent deviation of one parameter value. It is obvious that the influence of the deviation on the position of the local extremum $H(\boldsymbol{X}, f^*)$ is much greater than on the value of y_P determined by (21). When the local extrema of the frequency response are equalized, this is not observed. The sensitivity measurement error depends significantly on the alignment of local extrema. Therefore, if necessary, optimization of the frequency response curve should be carried out before the sensitivity analysis.

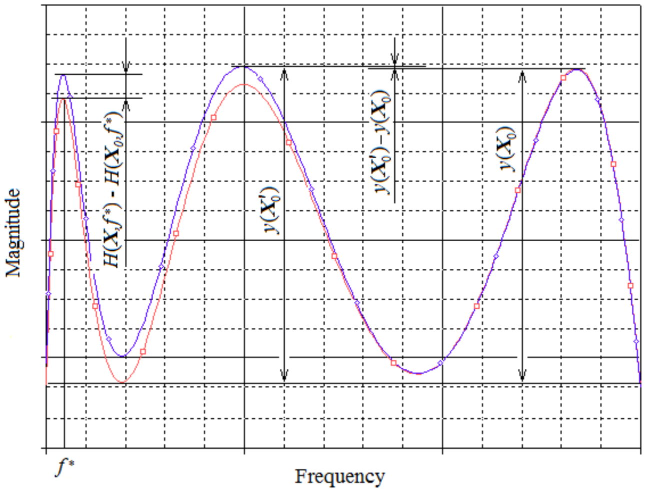

Fig. 2. Cause of sensitivity measurement error.

C. Op-amp Parameters

All real op-amps have a limited gain-bandwidth product (GBW). The limited bandwidth of the op-amp causes errors in the filter's frequency response. These errors can be reduced by performing parametric optimization for a specific GBW value. But GBW value deviations reduce the optimization efficiency. Therefore, when allocating tolerances for passive components, it is recommended to take into account the manufacturing variation of GBW. This is possible if the EDA tools are able to calculate the op-amp's GBW sensitivity.

In PSpice Advanced Analysis libraries, components can contain one or more tolerance parameters. Tolerance parameters for op-amp GBW could be GBW_POSTOL (positive tolerance) and GBW_NEGTOL (negative tolerance). Distribution parameters define types of distribution functions. Op-amp datasheet usually gives only the typical value for the GBW. Typically, the deviation can be ±30% of this value at room temperature, and an additional error of ±30% over the specified temperature range.

V. EXAMPLE

A. Tolerance allocation

A sixth-order Chebyshev bandpass filter is shown in Fig. 3. A schematic was created in OrCAD Capture. The frequency response specifications include a passband in the range of 500-2000 Hz, stopband edge frequencies are 200 Hz and 5000 Hz, the desired signal gain in the passband $H_P = 0$ dB, permissible ripple height $\Delta H_P = 1.8$ dB, attenuations in the stopband at least 35 dB. The filter design was performed for ripple $\Delta H_P(\boldsymbol{X}_0) = 1$ dB and $H_{SMAX}(\boldsymbol{X}_0) = -35.5$ dB. The operability margins were $a_{PMIN} = 0.4$ dB and $a_{SMIN} = 0.5$ dB. Multiple feedback stages are built using LM258A from the Advanced Analysis library. The E24 series values are assigned to C1-C6 capacitors. Frequency response errors caused by limited GBW have been reduced using the Optimizer Modified LSQ engine. The optimized resistance values are given in Table I.

Semi-relative sensitivities were found using the Sensitivity Analysis tool at 1% deviation of parameter values. The performance parameters defined in (21)-(22) and resistance values without rounding to the standard have been used. For capacitors C1-C6, tolerances of 1% are specified. For resistors, the tolerances were determined according to (20). GBW tolerances of 60% were taken into account. The E series are defined for the found tolerance values. Using the Discrete engine, standard resistance values were found. The tolerance allocation results are shown in Table I.

B. Tolerance Analysis

Verification of the tolerance allocation results was performed by using the Advanced Analysis Monte Carlo tool. Analysis results for 500 runs with calculated tolerance values for resistors and assigned tolerance values for capacitors and GBW are shown in Fig. 4. The residual operability margin reaches 0.07 dB in the passband.

Monte Carlo results for 500 runs with standard values of part parameters are shown in Fig. 5 and Fig. 6. After rounding, a yield of 99.6% in passband and 100% in stopband is achieved. The accuracy of the result obtained using the proposed technique can be considered acceptable.

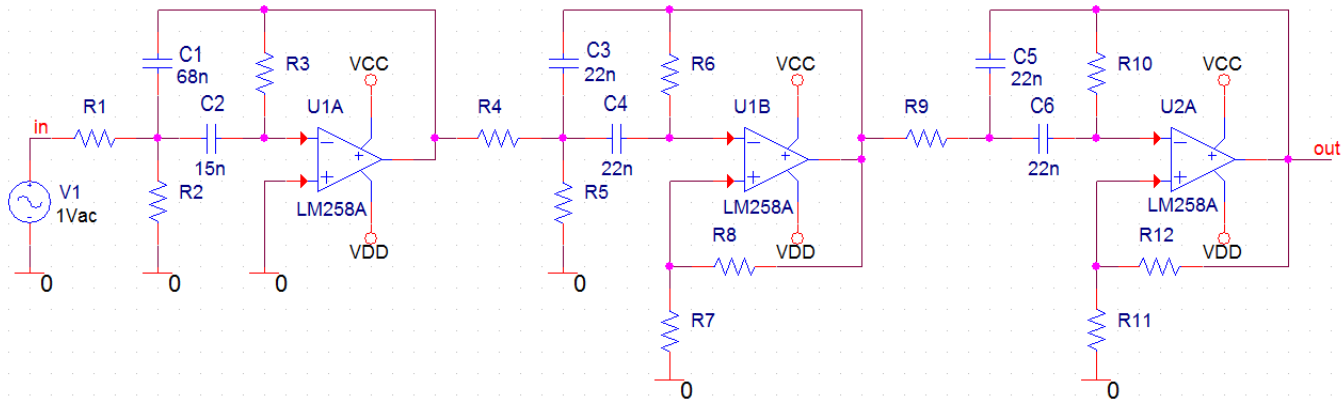

Fig. 3. Chebyshev bandpass filter.

TABLE I. TOLERANCE ALLOCATION RESULTS

Part	**Parameter**			
	Optimized value, kΩ	***Sensitivity, dB/%***	***Calculated (assigned) tolerance, %***	***Standard value, kΩ***
R1	2.976	−0.0533	0.586 (0.5)	2.98
R2	2.719	−0.0584	0.535 (0.5)	2.71
R3	17.46	−0.0899	0.347 (0.25)	17.4
R4	22.77	0.0283	1.102 (1)	23.2
R5	8.499	0.0789	0.396 (0.5)	8.45
R6	32.89	0.0313	0.998 (1)	33.2
R7	2,470	−0.0291	1.074 (1)	2.49
R8	10	0.145	0.216 (0.25)	10
R9	1.584	0.108	0.289 (0.25)	1.58
R10	8.443	0.226	0.138 (0.1)	8.45
R11	2.450	0.134	0.233 (0.25)	2.49
R12	10	−0.0335	0.933 (1)	10

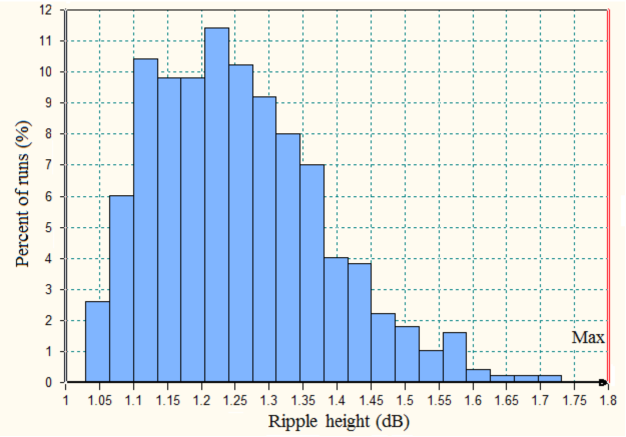

Fig. 4. Passband ripple height with exact parameter values.

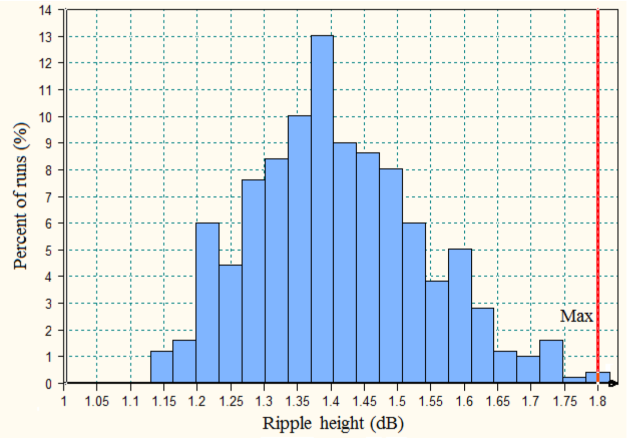

Fig. 5. Passband ripple height with standard parameter values.

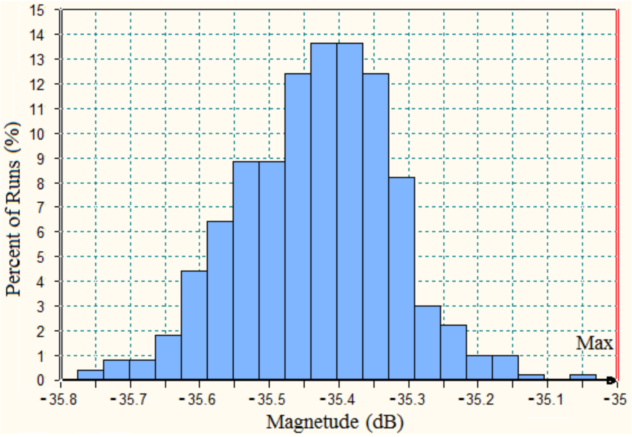

Fig. 6. Maximum stopband magnitude with standard parameter values.

CONCLUSION

- An engineering technique for active RC filters tolerance synthesis is considered. The technique based on sensitivity analysis assumes the use of advanced EDA means, including the measuring of performance parameters, tools for analyzing their sensitivities, libraries of components with tolerance parameters, and statistical analysis tool.
- Operability margins are the main resource in the tolerance allocation. The operability margins are determined by the permissible and actual values of the performance parameters, measured by the frequency response. Sensitivity values should be found of these performance parameters also.
- The availability of discrete resistors and capacitors with the required tolerances varies greatly. The proposed approach to the tolerance allocation based on the balance of variances, allows assigning available tolerance values to capacitors.
- It has been observed that deformation of the frequency response can cause gross errors when analyzing the sensitivity of the performance parameter using the OAT method. Before that, it is recommended to align the local extrema of the frequency response.

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