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# Conservative Grids Used for the Two-Dimensional Electromagnetic Analysis

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***Abstract* - Equivalent circuits of an elementary space cell that consist of lumped elements are proposed for constructing conservative grids used in the two-dimensional analysis of structures with arbitrary distributed permittivity and permeability. The derivation of the values of lumped elements is carried out on the basis of the finite-difference form of Maxwell's equations. In this case, all values of the obtained lumped capacitances and inductances take positive values, ensuring the computational stability and causality of the algorithms. It is demonstrated that the algorithms based on the grids obtained are numerically stable for positive, negative, and zero values of permittivity and permeability. When the values of the dielectric constant are lower than the dielectric constant of vacuum, including negative ones and equal to zero, the frequency dispersion in the proposed grid corresponds to the plasma model without taking into account the motion of positive ions and collisions, which makes it possible to use this model for the numerical electrodynamic analysis of an inhomogeneous plasma.**

***Keywords - conservative grids; electrodynamic analysis; non-uniform distribution of dielectric and magnetic permeability.***

## I. INTRODUCTION

In order to solve an analysis problem for a structure, one has to a space element with an operator, equivalent to the Maxwell equations. Similar operators for planar structures were constructed in [2], [4-6], [8-12]. However, the equivalent circuits corresponding to these operators should be completed so as to provide a stable computational procedure for the media with permittivity $\varepsilon < \varepsilon_0$ ($\varepsilon_0$ - is the permittivity of vacuum) as well as for the media with $\mu < \mu_0$ ($\mu_0$ - in the permeability of vacuum). The physical sense of the operator corresponding to a space element is most evident in the frequency domain (a so-called $RLC$- circuit consisting of lumped elements). so-called Therefore, first, we construct consisting a lumped – element circuit and, then, pass to a Rt – circuit consisting of transmission line segments and stubs, which facilitate the time – domain analysis.

## II. EQUIVALENT SCHEME OF THE ELEMENT OF SPACE FROM LUMPED ELEMENTS

As is know, the Maxwell equations can be written in the differential form as follows:

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{j}^e ; \ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{j}^m \tag{1}$$

For an isotropic medium, the constituent equations are:

$$\vec{D} = \varepsilon_a \vec{E} ; \ \vec{B} = \mu_a \vec{H} \tag{2}$$

In the case of monochromatic oscillations at circular frequency $\omega$

$$\frac{\partial \vec{D}}{\partial t} = i\omega \vec{D} ; \ \frac{\partial \vec{B}}{\partial t} = i\omega \vec{B} \tag{3}$$

Thus, taking into account conditions (2), (3) for the problem of two-dimensional analysis (the fields and geometry of a region under study are independent of one of the coordinates, for example, $z$) of $H$- polarized $(H_z = 0; E_x = E_y = 0)$ propagating in the absence of extremal sources ($\vec{j}^e = 0$; $\vec{j}^m = 0$) one can represent the Maxwell equations (1) in the Cartesian coordinates system in the from:

$$\frac{\partial E_z}{\partial y} = -i\omega\mu_a H_x ; -\frac{\partial E_z}{\partial x} = -i\omega\mu_a H_y ; \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = i\omega\varepsilon_a E_z \tag{4}$$

The system of equations (4) is often written in the form of the Helmholtz equation:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2\right) \cdot E_z = 0 \tag{5}$$

where $k^2 = \omega^2 \varepsilon_a \mu_a$, $k$ - wavenumber.

The system of equations (4) can be expressed in terms of finite differences

$$\frac{\Delta E_z}{\Delta y} = -i\omega\mu_a H_x ; \ -\frac{\Delta E_z}{\Delta x} = -i\omega\mu_a H_y ; \frac{\Delta H_y}{\Delta x} - \frac{\Delta H_x}{\Delta y} = i\omega\varepsilon_a E_z \tag{6}$$

System (6) can be solved directly, as is done in the finite difference method. Or, as shown in [1] - [3], [21], put in correspondence to system (6) the so-called $RLC$ the equivalent circuit of the space element shown in Fig. 1, which is equivalent to the two-dimensional telegraph equation.

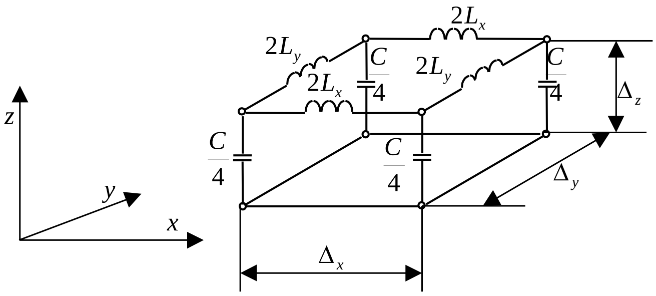


Fig. 1. Equivalent $RLC$ schema of a space element

In this case, the voltages $U$ applied to the capacitors $\frac{C}{4}$ are determined by the electric field intensities $E_z$. The currents $I_x$ and $I_y$ flowing through the inductances $2L_x$ and $2L_y$ determine the magnetic field inductances $H_x$ и $H_y$ respectively:

$$U = E_z \cdot \Delta_z ; I_x = H_y \cdot \Delta_y ; I_y = H_x \cdot \Delta_x \tag{7}$$

where $\Delta_x$, $\Delta_y$, $\Delta_z$ - are geometric dimensions of a space element (rectangular parallelepiped) along the axes $x$, $y$, $z$ respectively.

The rated values of elements of the equivalent circuit are determined by the parameters of the space element:

$$C = \varepsilon_a \frac{\Delta_x \cdot \Delta_y}{\Delta_z} ; L_x = \mu_a \frac{\Delta_x \cdot \Delta_z}{\Delta_y} ; \; L_y = \mu_a \frac{\Delta_y \cdot \Delta_z}{\Delta_x} \tag{8}$$

where $\varepsilon_a$ and $\mu_a$ are the absolute permittivity and permeability of the medium filling the space element.

The total geometry of the problem is assembled ("assembled") from elementary volumes by combining equivalent space elements (see Fig. 2).

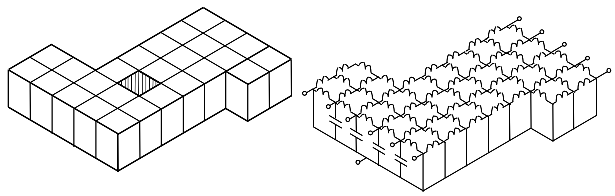

Fig. 2. Combining equivalent $RLC$ circuits of elementary volumes and the formation of an equivalent circuit of the entire analyzed geometry.

First, a complex domain under study is decomposed into simple units of the same type (Fig. 1). Then, the device is recomposed from unit elements. Thus, the electrodynamic problem is reduced to determining the parameters of a multiport, equivalent to the whole domain under study, i.e., to a problem of circuit theory. The boundary conditions are re specified by connecting different loads to the terminals of the multiport. This procedure is called the construction of the impedance model of the original electrodynamic problem. This allows us to pass from the language of differential equations (formal and mathematical) to the description of the problem in the language of impedance grids (schematic). Unlike the formal mathematical description, the circuit representation makes the opera-tions with the model rather evident.

This model enables one solve the problems of electrodynamic analysis for dielectric and magnetic media with $\varepsilon_a > \varepsilon_0$ and $\mu_a > \mu_0$, respectively. Since the formal inclusion of a negative capacitor in the circuit $C$ at $\varepsilon_a < 0$ ( or negative inductors $L_x$ and $L_y$ at $\mu_a < 0$ ) means actually connecting the generator to an equivalent circuit, then to obtain a computationally stable procedure for the electrodynamic analysis of media with $\varepsilon_a \le 0$ and $\mu_a \le 0$ it is necessary to modify the scheme of the space element (Fig. 1), and, hence, to modify the original system of equations (4).

Borrowing the representation of the electromagnetic space from [13- 16], we put the unfilled space into correspondence to a multiport (Fig. 2), formed by the recomposing to elementary volumes of space (Fig. 3).

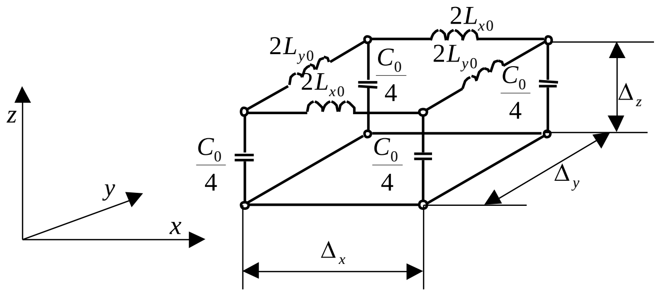


Fig. 3. Equivalent. $RLC$ scheme of an element of space not filled with matter.

The rated values of the elements of this equivalent circuit are found from (8):

$$C_0 = \varepsilon_0 \frac{\Delta_x \cdot \Delta_y}{\Delta_z} ; \; L_{x0} = \mu_0 \frac{\Delta_x \cdot \Delta_z}{\Delta_y} ; \; L_{y0} = \mu_0 \frac{\Delta_y \cdot \Delta_z}{\Delta_x} \tag{9}$$

where $\varepsilon_0$ - dielectric , $\mu_0$ - magnetic permeability of vacuum.

The medium is simulated by connecting one-port elements to the circuit obtained; i.e., the effect of the medium is local and the interaction is performed - through vacuum (Fig. 2). This mode1 is called an informative multiport model.

When the absolute permittivity of a space element under consideration differs from the permittivity $\varepsilon_0$ of vacuum, we connect the one-port elements У/4 to the circuit of the space element (Fig. 4).

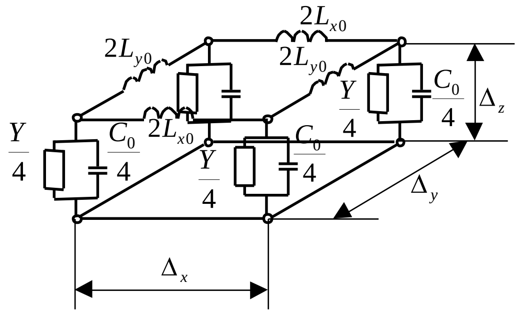


Fig. 4. Simulation of $RLC$ a diagram of a dielectric filling space element.

The parameters of the connected two-terminal networks must be such that at the analysis frequency $\omega$ the circuits shown in fig. 3 and fig. 1 were equivalent.

We will not introduce into the grid elements that absorb and generate energy. Let our grid be conservative. This means that the two-pole $Y/4$ will be either inductance or capacitance. Those. there are two cases to consider.

The first case corresponds to $\varepsilon_a > \varepsilon_0$, when

$$\varepsilon_a = \varepsilon_0 + \varepsilon^+ ;\; C = C_0 + C^+ ;\; C^+ = \varepsilon^+ \frac{\Delta_x \cdot \Delta_y}{\Delta_z} \tag{10}$$

In this case, the medium increases the capacitance $C_0$ of the space element by $C^+$. The equivalent circuit of the space element (Fig. 4) is transformed into the circuit in (Fig. 5).

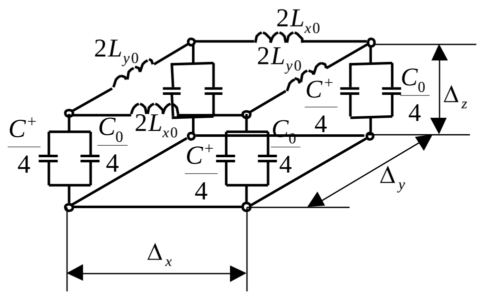


Fig. 5. Simulation of $RLC$ a diagram of a dielectric filling space element with a substance $\varepsilon_a > \varepsilon_0$.

The second case corresponds $\varepsilon_a < \varepsilon_0$:

$$\varepsilon_a = \varepsilon_0 - \varepsilon^- ;\; C = C_0 - C^- ;\; C^- = \varepsilon^- \frac{\Delta_x \cdot \Delta_y}{\Delta_z} ;\; L^- = \frac{1}{\omega^2 C^-} \tag{11}$$

In this case, the medium decreases the capacitance $C_0$ of the space element by $C^-$. This decrease in the capacitance is equivalent to including the inductance $L^-$ in the circuit, which implies that, when $\varepsilon_a < \varepsilon_0$ the medium brings an additional inductance, thus decreasing the capacitance $C_0$ of vacuum. The equivalent circuit of the space element (Fig. 4) is transformed into the circuit shown in Fig. 6 [7], [19], [20].

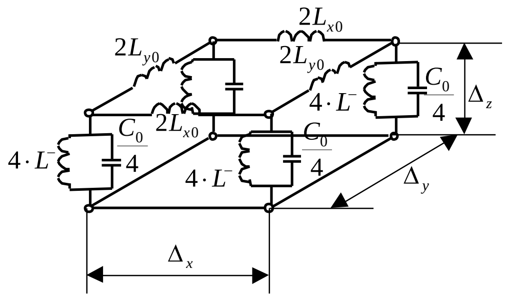


Fig. 6. Simulation of $RLC$ a diagram of a dielectric filling space element with a substance $\varepsilon_a < \varepsilon_0$.

This interpretation clearly illustrates the special cases $\varepsilon_a = 0$ and $\varepsilon_a < 0$.

When $\varepsilon_a = 0$ for parallel resonance is observed for parallel circuits formed by the inductances $4 \cdot L^-$ and containers parallel resonance is observed:

$$L^- = \frac{1}{\omega^2 C_0} \tag{12}$$

the oscillatory circuit is replaced by a gap.

When $\varepsilon_a < 0$ inductive component of the total admittance $\frac{1}{\omega \cdot 4 \cdot L^-}$ of the circuit exceeds the capacitive component $\omega \cdot C_0 / 4$, i.e.:

$$L^- < \frac{1}{\omega^2 C_0} \tag{13}$$

In both cases (12) and (13) considered above, the admittance of the circuit is zero or inductive. Therefore, waves do not propagate in the grid. The case $\varepsilon_a = 0$ is similar to the situation in a rectangular waveguide when the frequency of an exciting signal is equa1 to the critical frequency of the fundamental mode. The case $\varepsilon_a < 0$ - corresponds to the situation when the signal frequency is lower than the critical frequency of the fundamental mode.

In order to provide the existence of propagating modes on the impedance grid under consideration, one has to guarantee the condition $0 < \varepsilon_a$. In terms of the rated values of the equivalent circuit (Fig. 6) this means that:

$$L^- > \frac{1}{\omega^2 C_0} \tag{14}$$

In a similarly, one can simulate the space filled with a medium with permeability $\mu_a$, different from the permeability of vacuum $\mu_0$.

In this case, the medium is modeled by connecting to the circuit (Fig. 3) two-terminal $2 \cdot Z_x$ and $2 \cdot Z_y$ in series with inductors $2 \cdot L_{x0}$ and $2 \cdot L_{y0}$, so that at the frequency of analysis $\omega$ the circuits shown in fig. 7 and fig. 1 were equivalent.

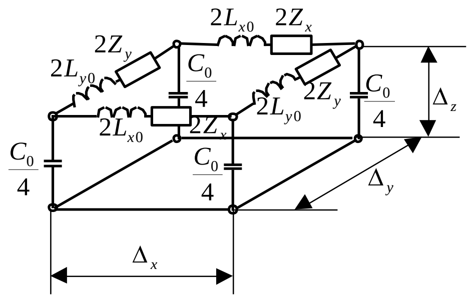


Fig. 7. Simulation of $RLC$ magnetic filling space element diagram.

Since the considered grid is conservative, the two-pole $2 \cdot Z_x$ and $2 \cdot Z_y$ - either inductance or capacitance. Therefore, it is necessary to consider two cases.

The second case corresponds to $\mu_a > \mu_0$:

$$\mu_a = \mu_0 + \mu^+; L_x = L_{x0} + L_x^+; L_y = L_{y0} + L_y^+; L_x^+ = \mu^+ \frac{\Delta_x \cdot \Delta_z}{\Delta_y};$$

$$L_y^+ = \mu^+ \frac{\Delta_y \cdot \Delta_z}{\Delta_x} \qquad (15)$$

In this case, the medium decreases the inductances $L_x^+$ and $L_{y0}$ of the space element by $L_x^+$ and an $L_y^+$ respectively. The equivalent circuit of the space element Fig. 7 is transformed into the circuit shown in (Fig.8).

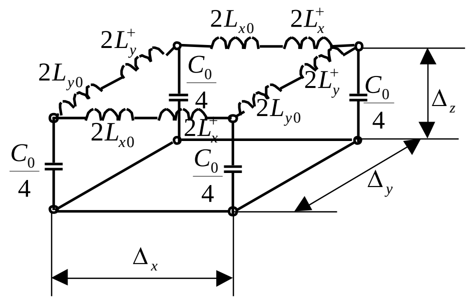


Fig. 8. Simulation of $RLC$ diagram of a magnetic filling space element with a substance $\mu_a > \mu_0$.

The second case corresponds to $\mu_a < \mu_0$, when

$$\mu_a = \mu_0 - \mu^-; \; L_x = L_{x0} - L_x^-; \; L_y = L_{y0} - L_y^-;$$

$$L_x^- = \mu^- \frac{\Delta_x \cdot \Delta_z}{\Delta_y}; \; L_y^- = \mu^- \frac{\Delta_y \cdot \Delta_z}{\Delta_x} \qquad (16)$$

$$C_x^- = \frac{1}{\omega^2 L_x^-}; \; C_y^- = \frac{1}{\omega^2 L_y^-} \qquad (17)$$

In this case, the medium reduces $L_{x0}$ and $L_{y0}$ f the space element by $L_x^-$ and an $L_y^-$ respectively. This decrease is equivalent to the presence of capacitances $C_x^-$ and $C_y^-$ (see Fig. 9).

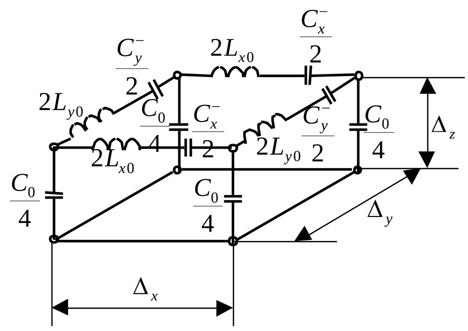


Fig. 9. Simulation of $RLC$ diagram of a magnetic filling space element with a substance $\mu_a < \mu_0$.

When $\mu_a < \mu_0$ the medium brings additional capacitances, thus decreasing the inductances $L_{x0}$ and $L_{y0}$ of vacuum. As a result, the equivalent circuit of the space element (Fig. 7) is transformed into the circuit shown in (Fig. 9).

Let us consider the special cases $\mu_a = 0$ and $\mu_a < 0$. When $\mu_a = 0$ a series resonance is observed for the series circuits formed by the inductances $2L_{x0}$ and capacitances $C_x^-/2$, and for contours consisting of $2L_{y0}$ and $C_y^-/2$:

$$C_x^- = \frac{1}{\omega^2 L_{x0}}; \; C_y^- = \frac{1}{\omega^2 L_{y0}} \qquad (18)$$

in this case, the oscillatory circuit is replaced by a short circuit.

When $\mu_a < 0$ the capacitive component of the total impedance of the circuit oriented along the $x$ $\frac{1}{\omega \cdot 2 \cdot C_x^-}$ ( for the circuit oriented a1ong the $y$ $\frac{1}{\omega \cdot 2 \cdot C_y^-}$) exceeds the inductive component $\omega \cdot 2 \cdot L_{x0}$ ($\omega \cdot 2 \cdot L_{y0}$), i.e:

$$C_x^- < \frac{1}{\omega^2 L_{x0}}; \; C_y^- < \frac{1}{\omega^2 L_{y0}} \qquad (19)$$

In cases (18) and (19), the circuits have zero or capacitive impedances. Therefore, in these cases, waves do not propagate in the grid.

ln order to provide the existence of propagating modes in the impedance grid, one has to fulfill the condition $0 < \mu_a$.

n terms of the rated values of the circuit (Fig. 9) this means that:

$$C_x^- > \frac{1}{\omega^2 L_{x0}}; \; C_y^- > \frac{1}{\omega^2 L_{y0}} \qquad (20)$$

In order to model the medium in the equivalent circuit of the space element when $\varepsilon_a \neq \varepsilon_0$ and $\mu_a \neq \mu_0$ it is necessary to combine the circuits presented ins Figs. 5, 6, 8 and 9, depending on $\varepsilon_a$ and $\mu_a$ (Fig.10 - 13).

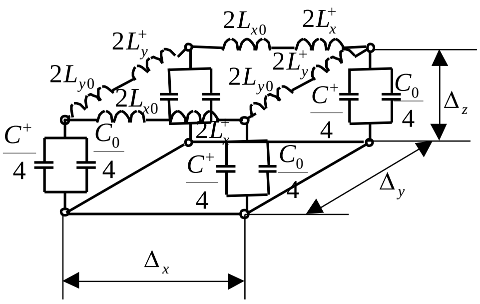


Fig. 10. Simulation of $RLC$ a diagram of a space element of dielectric and magnetic filling for a substance with $\varepsilon_a > \varepsilon_0$ и $\mu_a > \mu_0$.

The rated values of elements in the circuits shown in Fig. 10 - 13 are calculated using expressions (10), (11), (15) and (16).

Thus, conservative $RLC$ – circuits of a space element can be constructed for the analysis of the two-dimen-sional problem for $H$ - polarized waves when $\varepsilon_a \neq \varepsilon_0$ and $\mu_a \neq \mu_0$ [17], [18]. Assembling these circuits yields a conservative grid of the region under study, which provides the stability of the computational procedure for the electrodynamic ana1ysis.

## III. Conclusion

The proposed equivalent circuits of a space element made of lumped elements make it possible to form conservative grids for two-dimensional electromagnetic analysis of systems with an arbitrary distribution of dielectric and magnetic permeabilities in the frequency domain. The analysis algorithms built on the basis of such grids are numerically stable at positive, negative, and zero values of the permittivity and permeability.

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