In-Motion Calibration and Testing of MEMS Sensors Using a Reference Inertial Satellite Navigation System

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***Abstract*—A two-stage calibration of inertial micro-electro-mechanical (MEMS) sensors is considered. In this paper, such sensors are gyroscopes and accelerometers, oriented along three orthogonal axes and placed in a single inertial measurement module. The first stage of calibration is carried out in the factory at the bench without linear overloads. The second stage is implemented in a dynamic mode in a mobile laboratory that provides linear overloads. At this stage, the drifts of the sensor signals are estimated, as well as the skews of their measuring axes, which remained after the factory calibration. In addition, structural and parametric identification of dynamic models of sensor signal drifts is performed. Such models are used to compensate for the errors of MEMS sensors in autonomous inertial navigation modes, including the loss of satellite signals. The errors of MEMS sensors in motion are estimated using information from the reference inertial satellite navigation system and the extended Kalman filter. The results of full-scale experiments are analyzed.**

Keywords*—*inertial navigation system, global navigation satellite system, micro-electro-mechanical sensors, calibration, extended Kalman filter.

I. Introduction

The current state of onboard equipment of mobile objects is characterized by the use of integrated inertial satellite navigation systems (ISNS) [1]. In such ISNS global navigation satellite systems (GNSS) provide high-precision positioning, and inertial ones - the determination of the angular orientation and redundancy of the GNSS in case of failures. When limiting the size and mass of the ISNS, inertial navigation systems (INS) should be built on the basis of micro-electro-mechanical (MEMS) measuring modules. A typical MEMS module includes [2]: a triad of orthogonally placed gyros and a triad of orthogonally placed accelerometers. INS-MEMS built on the basis of the ADIS16488 measuring module developed by the Analog Devices Co [3] is shown in Fig. 1. A digital signal processor (DSP, see Fig. 1) based on the “OlinuXino A20 micro” computing board with an adapter intended for matching the SPI and UART interfaces.

It should be noted that inertial MEMS sensors have a wide insensitivity zone and low accuracy. Taking account of the above-mentioned features, INS-MEMSs must rely on the GNSS which forms part of an ISNS. Moreover, the INS-MEMSs cannot execute the initial alignment from attitude angles in the autonomous mode. Because of this, the initial alignment of INS-MEMS is realized from information obtained from the GNSS or reference INS.

Taking into account the prospects of using small-sized ISNS, as well as the capabilities of modern embedded computers, it seems expedient to develop analytical approaches to improving the accuracy of MEMS sensors. At the same time, when operating ISNS, the task is not only to estimate the deterministic errors of MEMS sensors, but also to identify dynamic models of their change.

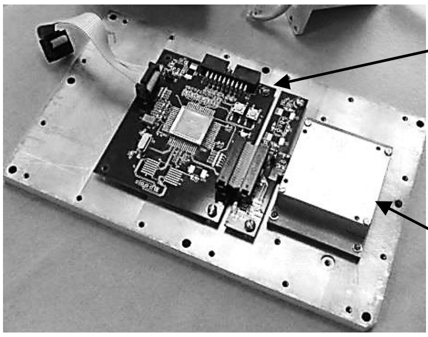

Fig. 1. INS-MEMS modules

Such models make it possible to maintain the required accuracy characteristics of the ISNS in an autonomous inertial mode when GNSS signals are lost. It should be noted that during bench calibration, it is not possible to simulate the dynamic modes of ISNS operation.

Traditionally [4] - [7] dynamic calibration of MEMS sensors in motion is performed using position and velocity data from GNSS. However, in this case, the errors of the gyros have poor observability. Therefore, it is proposed to use, in addition to velocity and positional parameters, angular information from the reference more accurate ISNS.

The purpose of this paper is to study the calibration procedures of MEMS sensors in motion along a given route using information from the reference ISNS.

The quality of calibration of MEMS sensors is estimated in post-processing of data recorded during a test trip along a given route.

II. Factory Calibration of MEMS Sensors

At bench calibration of a MEMS sensors the vector of errors, as a rule, includes [8], [9] systematic drifts of signals of sensitive elements (SE): gyroscopes and accelerometers, as well as angular deviations of the SE axes from an ideal orthogonal trihedron. For example, for a gyroscopic module, such a vector will have the form

$$x = [\, x_\omega^{\mathrm{T}} \, x_\delta^{\mathrm{T}} \,]^{\mathrm{T}}, \qquad (1)$$

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where $x_{\omega} = [\Delta\omega_x \Delta\omega_y \Delta\omega_z]^{\mathrm{T}}$ (2)

is the vector of systematic angular drifts of gyros;

$$x_{\delta} = [\delta_{xy} \delta_{xz} \delta_{yx} \delta_{yz} \delta_{zx} \delta_{zy}]^{\mathrm{T}} \quad (3)$$

is the vector of gyro angular skews; *ox*, *oy*, *oz* are the axes of the inertial measurement unit (IMU).

The calibration procedure is associated with the formation of observations, when the bench rotates sequentially around the axes *ox*, *oy*, *oz*. For example, when rotating around the *ox* axis of the IMU, the observation will have the form

$$Z_x = [\omega_x \omega_y \omega_z]^{\mathrm{T}}_{\mathrm{IMU}} - [\omega_x\ 0\ 0]^{\mathrm{T}}_{\mathrm{Bench}}. \quad (4)$$

Observation (4) can be associated with the following model,

$$Z_x = H_x x + \vartheta,$$

where $H_x = \begin{bmatrix} 1\ 0\ 0 & \omega_y & \omega_z & 0 & 0 & 0 & 0 \\ 0\ 1\ 0 & 0 & 0 & \omega_x & \omega_z & 0 & 0 \\ 0\ 0\ 1 & 0 & 0 & 0 & 0 & \omega_x & \omega_y \end{bmatrix}_{\mathrm{IMU}}$;

$\vartheta = [\Delta\omega_x \Delta\omega_y \Delta\omega_z]^{\mathrm{T}}_{\mathrm{Bench}}$ is the vector of errors in the formation of reference angular velocities of rotation on the bench.

During factory calibration, as a rule, bench errors are not taken into account. Then the estimate of the error vector of the inertial sensors $\hat{x}$ can be found using the least squares method by solving the following equation

$$\hat{x} = H^{-1}_{xyz} Z_{xyz}, \quad (5)$$

where $H_{xyz} = [H^T_x H^T_y H^T_z]^T$; $Z_{xyz} = [Z^T_x Z^T_y Z^T_z]^T$

It is possible to increase the accuracy of estimating the errors of inertial sensors by means of sequential modification of the extended Kalman filter (EKF) [10]. In this case, the unreliable operation of inverting the H_{xyz} matrix in equation (5) is excluded, and random errors of the rotary bench are also taken into account.

During bench calibration, it is not possible to perform the following procedures:

- estimation of the dynamic errors of the IMU arising from the influence of linear overloads;
- estimation of the dynamic errors of the IMU arising from the complex effect of linear and angular jerks;
- identification of dynamic models of sensor errors;
- quality control of the performed calibrations.

Before testing in motion, the factory calibration coefficients are stored in the INS-MEMS processor module.

III. Strapdown Inertial Satellite Navigation System as a Reference Object for Calibrating MEMS Sensors

Calibration of INS-MEMS sensors in motion can be performed by interacting with the reference ISNS. To do this, signals from inertial sensors of both systems, as well as GNSS signals, must be synchronously recorded. In post-processing, according to the registered data, the INS-MEMS solves the problem of inertial navigation, and in the reference ISNS, the problem of inertial-satellite navigation. In the work under consideration, the SINS-500 system [11], shown in Fig. 2, was used as a reference ISNS. The IMU of the SINS-500 system is based on fiber-optic gyros and quartz accelerometers. Fiber-optic gyros (FOG) were developed by the “Optolink” RPC (Zelenograd). The presence of the built-in flash-memory allows to develop the software and mathematical support according to the registered data.

In the considered INS systems, the modes of initial alignment and navigation are implemented. In the initial alignment mode, the initial conditions for reckoning the movement parameters are determined. The geodetic coordinates of the point of the initial alignment are assumed to be known. In ISNS, the approximate values of the initial orientation angles are determined by the analytical gyrocompassing method (AGC) [9]. This method is implemented using the signals of the INS sensors: gyros and accelerometers, which measure the projections of the angular velocity vector of the Earth's rotation and the acceleration of gravity on the IMU axes.

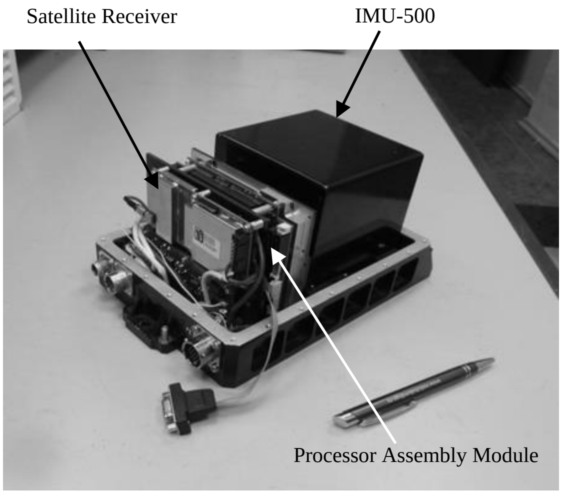

Fig. 2. SINS-500 strapdown inertial satellite navigation system.

The refinement of the orientation angles after the AGC mode, as well as the estimation of the instrumental drifts of the sensors, can be performed in the mode of fine initial alignment. This mode is implemented using observations of geophysical invariants that are both computed from INS information and “a priori” known. Typical are the invariants associated with the unmoved base of the SINS, namely:

$$Z_{\Theta(i)} = \int_{t_{i-1}}^{t_i} C^{\mathrm{T}}_0(\tau)\dot{\Theta}(\tau)\,d\tau - [0 \vdots 0 \vdots \Omega\Delta t_i]^{\mathrm{T}}; \quad (6)$$

$$Z_{c(i)} = [\varphi_i \lambda_i]^{\mathrm{T}}_{\text{INS-FOG}} - [\varphi_i \lambda_i]^{\mathrm{T}}_{\mathrm{PIA}}; \quad (7)$$

$$Z_{V(i)} = [V_\xi V_\eta V_\zeta]^{\mathrm{T}}_{(i)\text{INS - FOG}}, \tag{8}$$

where PIA stands for the position of the initial alignment; $\dot{\Theta} = [\dot{\Theta}_x \dot{\Theta}_y \dot{\Theta}_z]^{\mathrm{T}}$ is the vector of FOG output signals; Ω is the Earth's angular velocity; $\Delta t_i = t_i - t_{i-1}$ is an observation step; C_0 is the direction cosine matrix that characterizes the angular position of the IMU - fixed frame with respect to the inertial frame; φ, λ are the geodetic latitude and longitude of INS-FOG, which is the kernel of the ISNS; $\bar{V} = [V_\xi V_\eta V_\zeta]^{\mathrm{T}}$ is the vector of the relative velocity of IMU motion, given by its components along the axes of the semi-wander azimuth reference frame $o\xi\eta\zeta$ [11].

The INS-MEMS initial alignment is carried out according to the information of the INS-FOG.

Inertial-satellite navigation mode is implemented by processing the following observations using EKF

$$Z_{C(i)} = [\varphi_i \lambda_i]^{\mathrm{T}}_{\text{INS - FOG}} - [\varphi_i \lambda_i]^{\mathrm{T}}_{\text{GNSS}}; \tag{9}$$

$$Z_{V(i)} = C_1 [V_\xi V_\eta V_\zeta]^{\mathrm{T}}_{\text{INS - FOG}(i)} - [V_E V_N V_H]^{\mathrm{T}}_{\text{GNSS}(i)} \tag{10}$$

where C_1 is the direction cosine matrix that characterizes the angular position of the reference frame $o\xi\eta\zeta$ with respect to the geodetic frame *oENH*; *h* is the elevation with respect to the Earth ellipsoid.

In the quaternion implementation of the reckoning equations [11], the base vector of INS state includes errors in the reckoning of components of the relative velocity vector; errors in the reckoning of elements both of navigation and of attitude parameters; angular drifts of gyros, and biases of accelerometer signals.

IV. In-Motion Calibration of MEMS Sensors

When calibrating inertial sensors in motion, skews of the measuring axes of gyroscopes and accelerometers are additionally included in the INS-MEMS base state vector. Noise models of inertial sensors are formed so [8], [9] that their structure is mapped to the general error equation of INS

$$dx/dt = \dot{x}(t) = A(t)x(t) + G(t)\xi(t), \tag{11}$$

where $A(t) = \partial F[Y(t)]/\partial Y\big|_{Y(t) = Y_{\text{INS}}(t)}$ is the matrix of coefficients that characterize the dynamics of variation of INS errors; $\xi(t)$ is the vector of disturbances that affect the INS; $G(t)$ is the matrix of disturbance intensities; where $Y(t)$ is the motion parameter vector; $Y_{\text{INS}}(t)$ is the vector of parameters formed by the INS by solving the basic equations of inertial navigation. In general, these equations are nonlinear [8], [9] namely:

$$\dot{Y}_{\text{INS}}(t) = F[Y_{\text{INS}}(t)] + G(t)\xi(t); \tag{12}$$

$x(t) = Y_{\text{INS}}(t) - Y(t)$ is the vector of INS errors.

For the INS under consideration, equations (12) are presented in an expanded form in [2].

In calibration mode, the vector of INS-MEMS errors is estimated with the EKF by processing the following observations

$$Z_{C(i)} = [\varphi_i \lambda_i h_i]^{\mathrm{T}}_{\text{INS – MEMS}} - [\varphi_i \lambda_i h_i]^{\mathrm{T}}_{\text{ISNS}}; \tag{13}$$

$$Z_{V(i)} = [V_\xi V_\eta V_\zeta]^{\mathrm{T}}_{\text{INS - MEMS}(i)} - [V_\xi V_\eta V_\zeta]^{\mathrm{T}}_{\text{ISNS}(i)}; \tag{14}$$

$$Z_{\delta(i)} = [\psi \vartheta \gamma]^{\mathrm{T}}_{\text{INS - MEMS}(i)} - [\psi \vartheta \gamma]^{\mathrm{T}}_{\text{ISNS}(i)}, \tag{15}$$

where ψ is the true heading angle, ϑ is the pitch angle, and γ is the roll angle of the IMU.

When implementing observation (15), it is assumed that the mutual orientation of the inertial measurement modules IMU-MEMS and IMU-ISNS is known. After calibration, the angular misalignment values are stored in the INS-MEMS processor module. In the navigation mode, as a rule, only the basic INS-MEMS error vector is realized. The systematic angular drifts of the gyros and the biases of the accelerometer signals included in this vector have an autocorrelated character. In general, such a process can be described by the following equation

$$\Delta\mu(t) = \Delta\mu_s(t) + \xi(t), \tag{16}$$

where $\Delta\mu_s(t)$ is a systematic drift; $\xi(t)$ is a purely random error with a Gaussian distribution, i.e. $\xi(t) \in N(0, \sigma^2_\xi)$; σ^2_ξ is the variance of the perturbation. The perturbation $\xi(t)$ can be considered as an input signal for the error generation model.

In the process of calibration, it becomes necessary to identify the parameters of the models of autocorrelated sensor errors.

Equation (16) can be associated with the exponential correlation function

$$K_\mu(t) = E[\Delta\mu(t)\Delta\mu(t - T_\mu)] = \sigma^2_\mu e^{-|t|/T_\mu}, \tag{17}$$

where T_μ is the correlation time of the inertial sensor error; $E[\ldots]$ is the operator of mathematical expectation; $\mu = a$ is the index indicating the accelerometer; $\mu = g$ is the index indicating the gyroscope.

The spectral density of process (17) has the form

$$S(\omega) = \int_{-\infty}^{\infty} K_\mu(\tau) e^{-j\omega\tau} d\tau = \int_{-\infty}^{\infty} \sigma^2_\mu e^{-\alpha_\mu |\tau|} e^{-j\omega\tau} d\tau =$$
$$= 2\sigma^2_\mu \alpha_\mu / (\alpha^2_\mu + \omega^2), \tag{18}$$

where $\alpha_\mu = \dfrac{1}{T_\mu}$; $j = \sqrt{-1}$; ω is the frequency.

Spectral densities of the input $S_{\xi}(\omega)$ and output $S_{\Delta\mu}(\omega)$ signals in the inertial sensor model are related with module of frequency transfer function by the following ratio [12]:

$$S_{\Delta\mu}(\omega) = \left|W(j\omega)\right|^2 S_{\xi}(\omega). \quad (19)$$

For $\sigma_{\xi}^2 = 1$ the following relation is true

$$\left|W(j\omega)\right|^2 = 2\sigma_{\mu}^2\alpha_{\mu}/(\alpha_{\mu}^2 + \omega^2). \quad (20)$$

Taking into account relation (20), the transfer function in the Laplace form will have the following expression

$$W(p) = \sqrt{2\sigma_{\mu}^2\alpha_{\mu}}/(p + \alpha_{\mu}) = \Delta\mu(p)/\xi(p). \quad (21)$$

Hence, the equations describing the dynamics of errors of inertial sensors in the time domain will have the form

$$\Delta\dot{\mu}(t) = -\alpha_{\mu}\Delta\mu(t) + \xi(t)\sigma_{\mu}\sqrt{2\alpha_{\mu}}. \quad (22)$$

Equation (22) is easily mapped to the general model of INS errors (11)

$$\dot{x}_{\mu}(t) = A_{\mu}(t)x_{\mu}(t) + G_{\mu}(t)\xi(t), \quad (23)$$

where $A_{\mu}(t) = -\alpha_{\mu}$; $\alpha_{\mu} = \frac{1}{T_{\mu}}$; $G_{\mu}(t) = \sigma_{\mu}\sqrt{2\alpha_{\mu}}$.

From expression (17) it follows that $K_{\mu}(0) = \sigma_{\mu}^2$. Therefore, the identification problem can be reduced to determining the parameter α_{μ} in the model (17), which minimizes the quadratic function

$$F(\alpha_{\mu}) = \sum_{j=0}^{N}[\hat{K}_{\mu(j)} - \sigma_{\mu}^2 e^{-\alpha_{\mu}\tau_j}]^2 \to \min_{\alpha_{\mu}}, \quad (24)$$

where $\hat{K}_{\mu(j)}$ is the statistical correlation function, determined by the estimates $\hat{x}_j$ recorded during the operation of the INS, namely:

$$\hat{K}_{\mu(k)} = \frac{1}{N}\sum_{i=k+1}^{N+k}\tilde{x}_i\tilde{x}_{i-k}; \quad (25)$$

$\tilde{x}_i = \hat{x}_i - m_x$; $m_x = \frac{1}{N}\sum_{i=1}^{N}\hat{x}_i$; $k = \overline{0,N}$;

N is the number of retrospective samples of sensor signals;

$\tau_j = j\Delta t$; $\Delta t = t_i - t_{i-1}$; t_i are discrete moments in time.

Differentiating function (24) with respect to α_{μ} and equating the derivative to zero, we obtain

$$\frac{\partial F(\alpha_{\mu})}{\partial\alpha_{\mu}} = 2\left[\sum_{j=0}^{N}\hat{K}_{\mu(j)} - \sum_{j=0}^{N}\sigma_{\mu}^2 e^{-\alpha_{\mu}\tau_j}\right] \cdot$$
$$\cdot\sum_{j=0}^{N}\sigma_{\mu}^2\tau_j e^{-\alpha_{\mu}\tau_j} = 0. \quad (26)$$

Considering that for sensors $0 < \alpha_{\mu} < 1$ and the second factor in equation (26) does not affect the solution, we can write

$$\sum_{j=0}^{N}\hat{K}_{\mu(j)} = \sum_{j=0}^{N}\sigma_{\mu}^2 e^{-\alpha_{\mu}\tau_j}. \quad (27)$$

From equation (27) with $j = 0$, the estimate of the variance of the error is determined

$$\sigma_{\mu}^2 = \hat{K}_{\mu(0)}. \quad (28)$$

Then the normalized correlation functions will have the form

$$\hat{r}_j = \frac{\hat{K}_{\mu(j)}}{\sigma_{\mu}^2}. \quad (29)$$

For $\hat{K}_{\mu(j)} > 0$, equation (27) can be associated with an equivalent expression written in terms of the natural logarithm function, namely:

$$\sum_{j=0}^{N}\ln\hat{r}_j = -\alpha_{\mu}\sum_{j=0}^{N}\tau_j. \quad (30)$$

Hence, the estimate $\hat{\alpha}_{\mu}$ of the parameter α_{μ} will have the form

$$\hat{\alpha}_{\mu} = -\sum_{j=0}^{N}\ln\hat{r}_j \Big/ \sum_{j=0}^{N}\tau_j. \quad (31)$$

The change in the sign of the experimental correlation function (25) shows the inconsistency of the model (17) with the real estimates of the errors of inertial sensors. This discrepancy can be eliminated based on the following procedures for adaptive identification of model parameters:

- identification based on a limited sample of estimates, at which $\hat{K}_{\mu(j)} > 0$;
- correction of the structure of the correlation function model, taking into account the presence of estimates for which $\hat{K}_{\mu(j)} < 0$.

Correction of the structure of the correlation function model can be performed on the basis of a combination of transcendental functions, in particular, using the exponential-cosine function

$$K_{\mu}(t) = \sigma_{\mu}^2 e^{-\alpha_{\mu}|t|}\cos\beta_{\mu}t. \quad (32)$$

It can be shown that the correlation function (32) corresponds to the following spectral density:

$$S(\omega) = \frac{\sigma_\mu^2 \alpha_\mu (b^2 + \omega^2)}{b^4 + 2(\alpha_\mu^2 - \beta_\mu^2)\omega^2 + \omega^4}, \qquad (33)$$

where $\alpha_\mu = 1/T_\mu$; $b^2 = \alpha_\mu^2 + \beta_\mu^2$; T_μ is the correlation time.

Taking into account expression (19), spectral density (33) can be associated with a shaping filter with a transfer function

$$W(p) = \frac{\sqrt{2\sigma_\mu^2 \alpha_\mu}\,(p+b)}{p^2 + 2\alpha_\mu p + b^2} = \frac{\Delta\mu(p)}{\xi(p)}. \qquad (34)$$

Expression (34) corresponds to the following system of equations in the time domain

$$\Delta\dot{\tilde{\mu}}(t) = \Delta\mu(t) + \xi(t)\sigma_\mu\sqrt{2\alpha_\mu}; \qquad (35)$$

$$\begin{aligned}\Delta\dot{\mu}(t) &= -b^2\Delta\tilde{\mu}(t) - 2\alpha_\mu\Delta\mu(t) + \\ &+ \xi(t)\sigma_\mu\sqrt{2\alpha_\mu}\,(b - 2\alpha_\mu).\end{aligned} \qquad (36)$$

These equations can be written in vector-matrix form

$$\underbrace{\begin{bmatrix}\Delta\dot{\mu}(t)\\ \Delta\dot{\tilde{\mu}}\end{bmatrix}}_{\dot{x}_\mu(t)} = \underbrace{\begin{bmatrix}-2\alpha_\mu & -b^2\\ 1 & 0\end{bmatrix}}_{A_\mu(t)} \underbrace{\begin{bmatrix}\Delta\mu(t)\\ \Delta\tilde{\mu}(t)\end{bmatrix}}_{x_\mu(t)} + \underbrace{\begin{bmatrix}(b-2\alpha_\mu)\sigma_\mu\sqrt{2\alpha_\mu}\\ \sigma_\mu\sqrt{2\alpha_\mu}\end{bmatrix}}_{G_\mu(t)}\xi(t)$$

In equations (35) and (36), parameter $\Delta\tilde{\mu}(t)$ is an auxiliary variable, and $\Delta\mu(t)$ is the total sensor error. In equations (35) and (36), the identified parameters α_μ and β_μ are determined by minimizing the quadratic function

$$F(\alpha,\beta) = \sum_{j=0}^{N} [\hat{K}_{(j)} - \sigma^2 e^{-\alpha|\tau_j|} \cos\beta\tau_j]^2 \to \min_{\alpha,\beta} \qquad (37)$$

at $\hat{K}_{(j)} := \hat{K}_{\mu(j)}$; $\alpha := \alpha_\mu$; $\beta := \beta_\mu$.

In the above problem, the parameters β_μ and σ_μ^2 are determined by the experimental correlation function $\hat{K}_{\mu(j)}$ exactly, namely:

$$\hat{\sigma}_\mu^2 = \hat{K}_\mu(0); \; \hat{\beta}_\mu = 0.5\pi/\tau_\beta, \qquad (38)$$

where τ_β is the value of the time interval for which $K(\tau_\beta) = 0$.

The following approaches to solving problem (37) are possible taking into account estimates (38):

- in the general case, through the decomposition of transcendental functions in power series;
- through the function of the natural logarithm for the sample of the estimates, for which $\hat{K}_{\mu(j)} > 0$, namely:

$$\left.\frac{\partial F(\alpha)}{\partial \alpha}\right|_{\alpha = \hat{\alpha}_\mu} = 2\sum_{j=0}^{N} (\ln \hat{r}_{\mu(j)} + \alpha\tau_j - \ln\cos\hat{\beta}_\mu\tau_j)\,\tau_j = 0$$

From this equation, an estimate of the parameter $\hat{\alpha}_\mu$ is obtained

$$\hat{\alpha}_\mu = \sum_{j=1}^{N} (\ln\cos\hat{\beta}_\mu\tau_j - \ln\hat{r}_{\mu(j)}) / \sum_{j=1}^{N}\tau_j,$$

where $\tau_j = j\Delta t_j$; $\Delta t_i = t_i - t_{i-1}$; $\hat{r}_{\mu(j)} = \hat{K}_{\mu(j)}/\sigma_\mu^2$.

After identification, parameters $\hat{\alpha}_\mu$, $\hat{\beta}_\mu$, and $\hat{\sigma}_\mu^2$ are included in equations (35) and (36).

In Fig. 3, typical results of the identification of normalized correlation functions of the angular drift of one of the gyros are shown, namely: for the function taken into

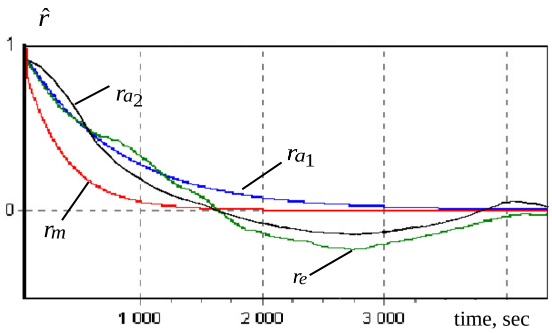

Fig. 3. Results of identification of normalized correlation functions.

account in the model, rm; for the function computed from experimental data, re; for the function obtained either by the exponential approximation, ra_1, or by the exponential cosine approximation, ra_2.

V. Analysis of the Results of Studies

Experiments have been carried out under terrestrial conditions when the necessary equipment was housed in a mobile laboratory. The timing diagram of operation of the SINS-500 and INS-MEMS systems included the following stages: initial alignment by the AGC method (t = 0÷270sec); fine initial alignment using invariants and EKF (t = 270÷740sec), and a navigation mode (t >740sec).

In the structure of the ISNS, the basic SINS-500 system operates in the indicator mode [13]. In this mode, error estimates are compensated in the output signals of the SINS. The use of smoothed data from the corrected SINS allows continuous estimation of MEMS sensor errors during the calibration process. In addition, in the SINS-500 system, monitoring and protection of the information integrity of inertial satellite observations is carried out [14].

Fig. 4 shows the circular positional error ΔS of the basic SINS-500 system in inertial satellite mode, where

$$\Delta S = \sqrt{\delta_{\varphi}^{2} + \delta_{\lambda}^{2}}\ ; \qquad (39)$$

$$\delta_{\varphi} = (\varphi_{\text{INS}} - \varphi_{\text{GNSS}})R;$$

$$\delta_{\lambda} = (\lambda_{\text{INS}} - \lambda_{\text{GNSS}})R\cos\varphi_{\text{GNSS}};$$

R is the value of the radius vector of ISNS position.

The value of parameter (39) is taken as a quality criterion for the calibration, which is performed iteratively using the recorded data.

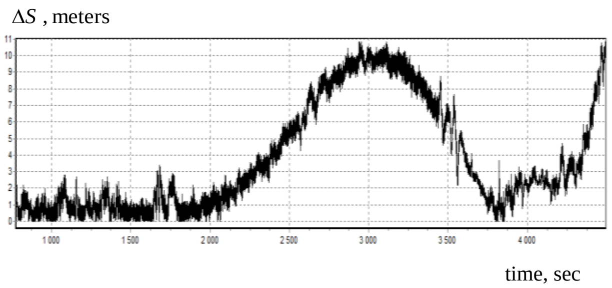

Fig. 4. Circular position error of the basic SINS-500 system in inertial satellite mode.

Fig. 5 shows the circular positional error of the INS-MEMS in the inertial mode after pre-calibration under stationary conditions.

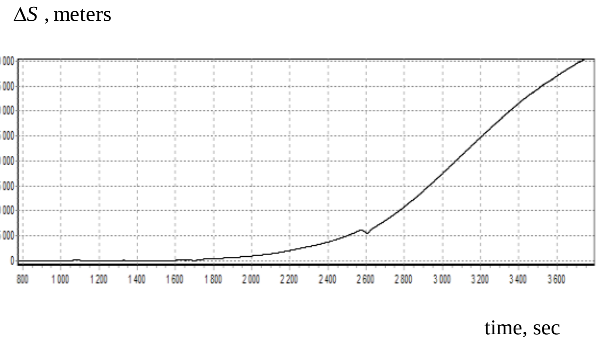

Fig. 5. Circular position error of the INS-MEMS in inertial mode after pre-calibration under stationary conditions.

Fig. 6 shows the INS-MEMS circular positional error in inertial mode after pre-calibration under stationary conditions and in motion.

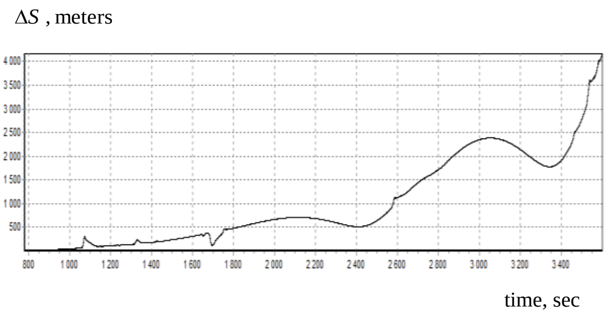

Fig. 6. Circular position error of the INS-MEMS in inertial mode after pre-calibration under stationary conditions and in motion.

It can be seen that after calibration in the dynamic mode, the circular positional error of the INS-MEMS decreased by an order of magnitude. It should be noted that the results were obtained without taking into account the thermal drifts of the MEMS sensors.

Conclusions

The conducted studies have shown the feasibility of performing a combined ground-onboard calibration of inertial measuring modules based on MEMS sensors. The proposed technology for such a calibration is based on the use of a reference inertial-satellite navigation system and the mathematical apparatus of the EKF. In addition, in the process of dynamic calibration, the structural and parametric identification of the error models of MEMS sensors, which are necessary for integration with the GNSS, can also be performed. In practice, the information from the reference ISNS can be used to calibrate several measurement modules based on MEMS sensors.

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